LOGIC EXERCISES: WEEK 8

QUESTION 1

Translate the following $L_=$-sentences into unambiguous English using the given dictionary.

Domain of discourse: people in Star Wars.

a: Luke	P: ... is heroic
b: Ben	Q: ... is a Jedi
c: Vader	R: ... is a Wookie
	S: ... is stronger in the Force than ...

i. ∃x∃y((Rx ∧ Ry) ∧ ¬x=y)
ii. ∀x∀y((Qx ∧ Qy) → x=y)
iii. (Pa ∧ ∀x(Px → x=a))
iv. ∀x(Qx ↔ x=a)
v. ∀x(Qx → x=a)
vi. ∃x((Px ∧ Rx) ∧ ∀y((Py ∧ Ry) → y=x) ∧ Qx)
vii. ∀x∃y(Sxy ∧ ¬x=y)
viii. ∃x∀y(¬x=y → Sxy)
ix. ∀x(x=c → Sxb)
x. ∀x((x=a ∨ x=b) → (Px ∧ Qx))

QUESTION 2

Using the same dictionary, formalize the following English claims in $L_=$.

i. There is only one Wookie.
ii. There is at most one Wookie.
iii. There are exactly two Wookies.
iv. There are at most two Wookies.
v. If no Wookies are heroic, then no Wookies are heroic Jedi.
vi. Vader is stronger in the Force than any Wookie.
vii. Any Wookie who is stronger in the Force than Ben is a heroic Jedi.
viii. No two Jedi are both stronger in the Force than Vader.
ix. No two Jedi are both stronger in the Force than each other.
x. If no Jedi is heroic, then Vader is stronger in the Force than anyone else.
xi. If any Wookie is stronger in the Force than some Jedi, then that Wookie is heroic.
xii. The Wookie who is stronger in the Force than Vader is heroic.
xiii. Only Luke and Ben are stronger in the Force than Vader.
xiv. Luke is only stronger in the Force than a Wookie.
xv. Only Luke is only stronger in the Force than a Wookie.

QUESTION 3

Produce $L_=$-structures that are counterexamples to the following incorrect claims of validity.

    i.   ∃x∃y((Gx ∧ Gy) ∧ ¬x=y), ∀x(∃y(Gy ∧ x=y) ∧ ¬Fx) ⊨ ∃x∃y((Fx ∧ Fy) ∧ ¬x=y)

    ii.   ∃xFx, ∀x∀y∀z(((Fx ∧ Fy) ∧ Fz) → ((x=y ∨ y=z) ∨ x=z)) ⊨ ∃x∀y(Fy ↔ x=y)

    iii.   Pab, Pba ⊨ ¬a=b

    iv.   ∀x∀y(Px → (Py → ¬x=y)) ⊨ ∃x∃y¬x=y

    v.   ∀x∀y(¬x=y → (¬Rxy ∨ ¬Ryx)) ⊨ ∀xRxx

QUESTION 4

Construct proofs in natural deduction to show that the following sequents are correct.

    i.   ⊢ ∃y y=y

    ii.   ∃xPx, ∃x¬Px ⊢ ∃x∃y ¬x=y

    iii.   ∃x(∀y(Py → x=y) ∧ Px) ⊢ ∃x∀y(Py ↔ x=y)

    iv.   ∃x∀y(Py ↔ x=y) ⊢ ∃x(∀y(Py → x=y) ∧ Px)